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**Question Paper Code : 20751**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to All Branches)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary constants  $a$  and  $b$  from  $z = (x - a)^2 + (y - b)^2 + 1$ .
2. Find the complete integral of  $p + q = x + y$ .
3. Write the complex form of Fourier series in the interval  $0 < x < 2\pi$ .
4. Find the Root mean square value of the function  $f(x) = x - x^2$  in  $-1 < x < 1$ .
5. Solve  $yu_x + xu_y = 0$  using separation of variables method.
6. What are the possible solutions of the one dimensional heat flow equation?
7. State Fourier integral theorem.
8. Prove that  $F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$ ,  $a > 0$ .
9. Find  $Z\left[\cos\left(\frac{n\pi}{2}\right)\right]$ .
10. State initial and final value theorem for Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary functions  $f$  and  $g$  from  $z = f(ax + by) + g(\alpha x + \beta y)$ . (8)

(ii) Find the general solution of  $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ . (8)

Or

(b) (i) Solve  $z^2(p^2 + q^2) = x^2 + y^2$ . (8)

(ii) Find the general solution of  $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$ . (8)

12. (a) (i) Find the Fourier series expansion of  $f(x) = x + x^2$  in  $-\pi < x < \pi$  and hence deduce the value of  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty$ . (8)

(ii) Find the Fourier series expansion of  $f(x) = 2x - x^2$  in  $0 < x < 3$ . (8)

Or

- (b) (i) Find the Fourier cosine series expansion of  $f(x) = x \sin x$  in  $0 < x < \pi$  and hence deduce the value of  $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} + \dots \infty$ . (8)

- (ii) Compute the first two harmonics of the Fourier series of  $f(x)$  from the table: (8)

$x$	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$f(x)$	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

13. (a) A string is stretched tightly between  $x = 0$  and  $x = 20$  is fastened at both ends. The midpoint of the string is taken to a height and then released from rest in that position. Find the displacement of any point  $x$  of the string at any time  $t$ . (16)

Or

- (b) A square plate is bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x = 20$  and  $y = 20$ . Its faces are insulated. The temperature along the upper horizontal edge is given by  $u(x, 20) = x(20 - x)$  when  $0 < x < 20$  while the other three edges are kept at  $0^\circ\text{C}$ . Find the steady state temperature in the plate. (16)

14. (a) (i) Find the Fourier Transform of  $f(x)$  if  $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  and hence

evaluate the integral  $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$ . (10)

- (ii) State and prove convolution theorem for Fourier transforms. (6)

Or

(b) (i) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$  using transforms. (6)

(ii) Find the Fourier cosine transform of  $f(x) = e^{-a^2x^2}$  and hence find  $F_S[xe^{-a^2x^2}]$ . (10)

15. (a) (i) Find  $Z\left[\frac{1}{(n+1)(n+2)}\right]$ . (8)

(ii) Using convolution theorem evaluate  $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ . (8)

Or

(b) (i) Using  $Z$  - Transform solve  $y(n+3) - 3y(n+1) + 2y(n) = 0$ , with  $y(0) = 4, y(1) = 0, y(2) = 8$ . (8)

(ii) Find  $Z^{-1}\left[\frac{z}{(z-1)(z^2+1)}\right]$  by using integral method. (8)

